Spike-Timing-Dependent-Plasticity (STDP) models or how to understand memory.

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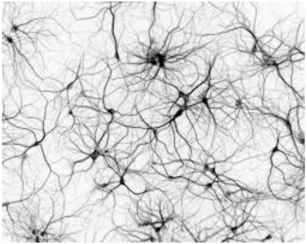
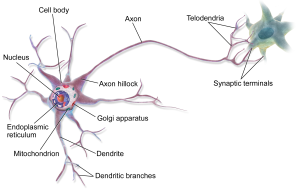
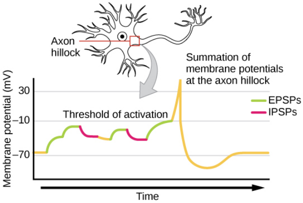
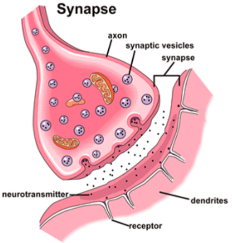
Neuron network functioning



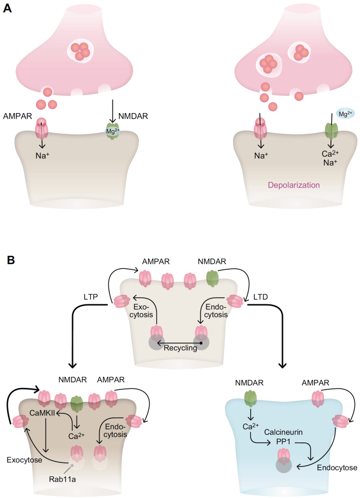




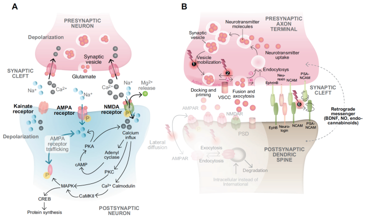


Biophysical Models

Basic cellular processes :



Pathways mediating functional synaptic plasticity :



Too complex

$\Rightarrow$

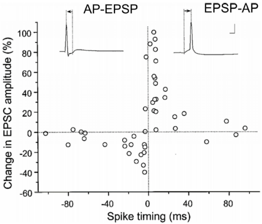
Phenomenological models

Main idea of STDP

- Hebb's law (1949) :

  "*Neurons that fire together, wire together.*" [?]

- Spiking-Time Dependent Plasticity (STDP)
  [Bi and Poo, 1998]



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Phenomenological STDP models

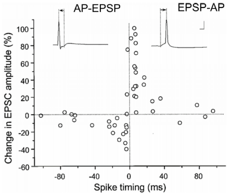
- Simplify biophysical models
- Neuron model
- Plasticity model
- Neuron network
- Simulations

Plasticity rule

STDP learning rule

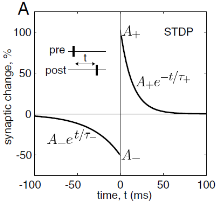
$$\Delta w = F_+(w)G(\Delta t_+) - F_-(w)G(\Delta t_-)$$

STDP experimental curve :



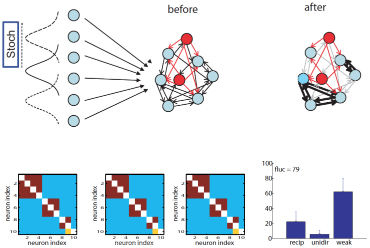
[Bi and Poo, 1998]

STDP deterministic curve (G) :



[Izhikevich and Desai, 2003]

Simulations



[Clopath et al., 2009]

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Objectives

- Rich enough to reproduce biological phenomena
- Simple enough to be analysed mathematically
- Take into account time scales differences
- Adapted to simulation of not too small networks (10 000 neurons)
- Observe global properties of the network

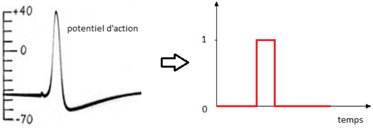
Individual neuron model

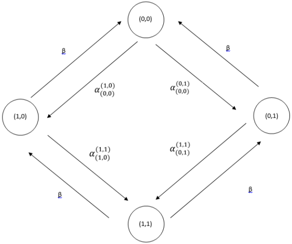
- Neurons are either at rest(0) or spiking(1).
- $W_t \in \mathbb{R}^N \times \mathbb{R}^N$ synaptic weights matrix.
- $V_t \in \{0,1\}^N$ neuron system state.
- Inhomogeneous jump rates :

$$0 \underset{\beta}{\overset{\alpha_i'(W_t, V_t)}{\rightleftharpoons}} 1$$

- $S : \mathbb{R} \mapsto \mathbb{R}_*^+$ bounded, positive and nondecreasing, $\alpha_m > 0$:

$$\alpha_i'(W_t, V_t) = S\left(\sum_{i=1}^{N} W_t^{ij} V_t^j\right) + \alpha_m$$





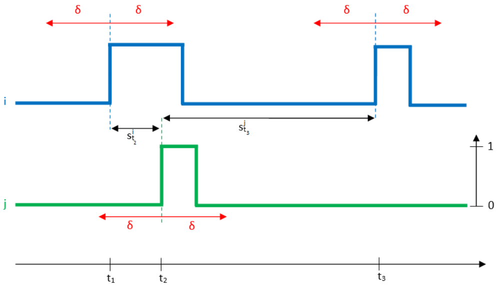
System with 2 neurons

Plasticity rule

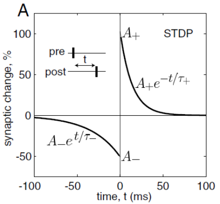
- Jump of Δw = cste

If the neuron i spikes at t^-, $\forall j \neq i$:

- if $s_t^j < \delta$ then $w_{ij} + \Delta w$ with probability $p^+(s_t^j) = 1_{s_t^j < \delta} A_+ e^{-\frac{s_t^j}{\tau_+}}$;
- if $s_t^j < \delta$ then $w_{ji} - \Delta w$ with probability $p^-(s_t^j) = 1_{s_t^j < \delta} A_- e^{-\frac{s_t^j}{\tau_-}}$.



STDP curve :

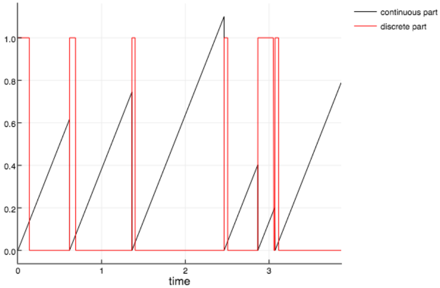


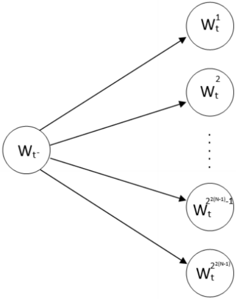
[Izhikevich and Desai, 2003]

Markov Process $(W_t, S_t, V_t)_{t>0}$ with $(W_0, S_0, V_0) = (w_0, s_0, v_0)$ and :

- $W_t \in \mathbb{R}^N \times \mathbb{R}^N$ synaptic weight matrix ;
- $S_t = (S_t^1, ..., S_t^N) \in \mathbb{R}^N$ inter-arrival time between spikes ;
- $V_t \in \{0, 1\}^N$ neuron system state.

State space : $E = (\mathbb{R}^N \times \mathbb{R}^N) \times \mathbb{R}^N \times \{0, 1\}^N$





$$G_i = \left\{ w + \Delta w \left(\underbrace{\begin{bmatrix} 0 & \dots & 0 \\ & \vdots & \\ 0 & \dots & 0 \\ & \vec{\gamma^i} & \\ 0 & \dots & 0 \\ & \vdots & \\ 0 & \dots & 0 \end{bmatrix}}_{N \times N \ \ matrix} + \begin{bmatrix} 0 & \dots & 0 & & 0 & \dots & 0 \\ \vdots & & \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & \vec{\zeta^i} & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots & & \vdots \\ 0 & \dots & 0 & & 0 & \dots & 0 \end{bmatrix} \right), \ (\vec{\gamma^i}, \vec{\zeta^i}) \in F^i \right\}$$

$$F^i = \left\{ (\vec{\gamma^i}, \vec{\zeta^i}), \vec{\gamma^i} = (\gamma^i_1, ..., \gamma^i_N), \vec{\zeta^i} = \begin{bmatrix} \zeta^i_1 \\ \vdots \\ \zeta^i_N \end{bmatrix}, \gamma^i_j, \zeta^i_j \in \{0,1\}, \gamma^i_i = \zeta^i_i = 0 \right\}$$

For $(\vec{\gamma^i}, \vec{\zeta^i}) \neq (0,0)$:

$$\phi(s, \vec{\gamma^i}, \vec{\zeta^i}) = \prod_{j \neq i} \left[\gamma^i_j p^+(s_j) + (1 - \gamma^i_j)(1 - p^+(s_j)) \right] \left[(\zeta^i_j p^-(s_j) + (1 - \zeta^i_j)(1 - p^-(s_j)) \right]$$

Generator of the process $(W_t, S_t, V_t)_{t>0}$

$$
\begin{aligned}
\tilde{C}f(w,s,v) = {} & \underbrace{\sum_i \delta_1(v^i)\beta[f(w,s,v-e_i) - f(w,s,v)]}_{B_\downarrow f(w,s,v)} \\
& + \phi(s,w,w)\underbrace{\sum_i \alpha_i(w,v)\delta_0(v^i)\left(f(w,\ s-s_ie_i,\ v+e_i) - f(w,s,v)\right)}_{B_\uparrow f(w,s,v)} \\
& + \underbrace{\sum_{i=1}^{N} \partial_{s_i} f(w,s,v)}_{B_{tr} f(w,s,v)} \\
& + \underbrace{\sum_i \alpha_i(w,v)\delta_0(v^i)\left(\sum_{\tilde{w}\in G_i,\tilde{w}\neq w} (f(\tilde{w},\ s-s_ie_i,\ v+e_i) - f(w,\ s,\ v))\phi(s,\tilde{w},w)\right)}_{\tilde{A}f(w,s,v)}
\end{aligned}
$$

Assumption on time scales

$$\sum_{(\vec{\gamma^i},\vec{\zeta^i})\neq(0,0)} \phi(s,\vec{\gamma^i},\vec{\zeta^i}) \ll \phi(s,0,0)$$

$\Rightarrow (S_t, V_t)_{t>0}$ change fast compare to $(W_t)_{t>0}$

First results

- W_t fixed, $(S_t, V_t)_{t>0}$ has a unique invariant measure π_{W_t}
- One can compute the Laplace transform of π_{W_t}
- Slow fast analysis gives the limit model for the weights :

$$Cf(w) = \int_{\mathbb{R}_+^2\times\{0,1\}^N} Af(w,s,v)\pi_w(ds,dv)$$

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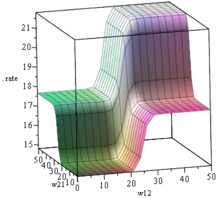
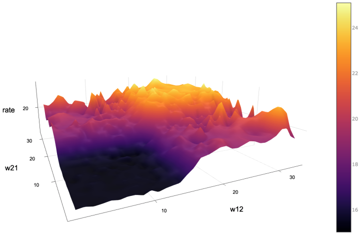
Simulations

2 neurons

Analytic



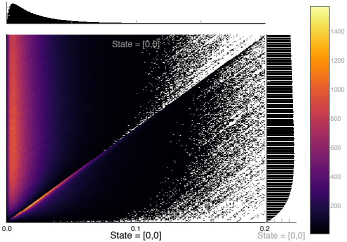
Numerical

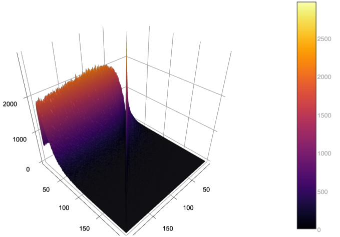


Simulations

2 neurons

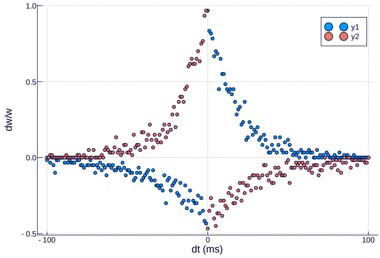
Discontinuity of the density

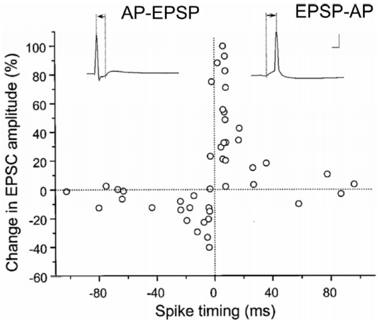




Simulations
N neurons

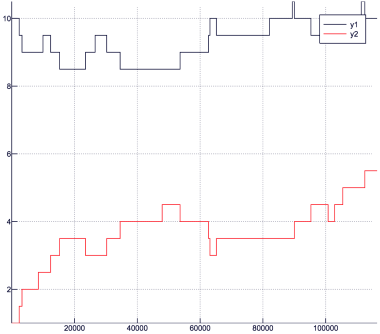
STDP curve

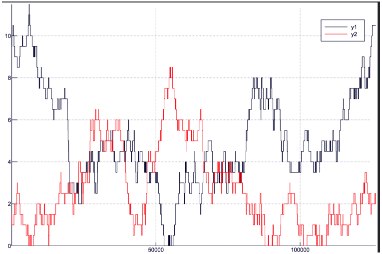




Simulations
2 neurons

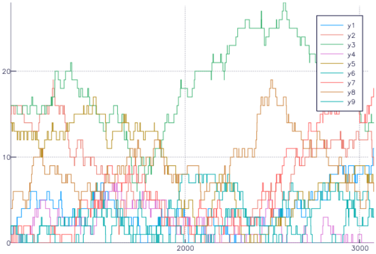
Trajectories





Simulations

10 neurons



Perspectives

- Study the weights dynamics
- Simulations to test with other plasticity rules
- Neurons states from discrete to continuous.
- Mean field approximations.

Bi, G.-q. and Poo, M.-m. (1998). Synaptic modifications in cultured hippocampal neurons : dependence on spike timing, synaptic strength, and postsynaptic cell type. *Journal of neuroscience*, 18(24) :10464–10472.

Clopath, C., Büsing, L., Vasilaki, E., and Gerstner, W. (2009). Connectivity reflects coding : A model of voltage-based spike-timing-dependent-plasticity with homeostasis. *Nature*.

Izhikevich, E. M. and Desai, N. S. (2003). Relating stdp to bcm. *Neural computation*, 15(7) :1511–1523.

Morrison, A., Diesmann, M., and Gerstner, W. (2008). Phenomenological models of synaptic plasticity based on spike timing. *Biological Cybernetics*, 98(6) :459–478.

Thank you for your attention

Do you have questions ?